Strain gauge and rosettes

Introduction

A **strain gauge** is a device which is used to measure strain (deformation) on an object subjected to forces.

Strain can be measured using various types of devices classified depending upon their principle of operation. Some of them are as follows:

1. Mechanical type,

2. Optical type

3. Pneumatic type

4. Electrical type

Earlier, mechanical type of device such as extensometer or extension meter was used to measure strain by measuring change in length.

Photoelectric strain gauge was also introduced which uses a light beam to produce electric current corresponding to deformation.

The most commonly used strain gauge is an **electrical resistance strain gauge**.

This strain gauge works on the principle that when a metallic wire type gauge is strained (here due to forces on object in contact), the resistance of the wire will be changed due to changes in its length, diameter and resistivity.

$$\text{Resistance (R)} = \frac{\rho L}{A}$$

Where ρ is resistivity; L is length of wire; A is area of crossection of wire.

This change in resistance will be in proportion with the strain produced which can be easily measured using Wheatstone bridge.

Drawbacks of Strain gauge

1. A strain gauge is capable only of measuring strain in the direction in which gauge is oriented.

2. There is no direct way to measure the shear strain or to directly measure the principal strains as directions of principal planes are not generally known.

Strain rosettes

Since for strain analysis in biaxial state of stress we should know strain in three directions and due to drawbacks in a strain gauge, Strain rosettes came in to picture.

Strain rosette can be defined as the arrangement of strain gauges in three arbitrary directions.

These strain gauges are used to measure the **normal strain** in those three directions.

Depending on the arrangement of strain gauges, strain rosettes are classified in to:-

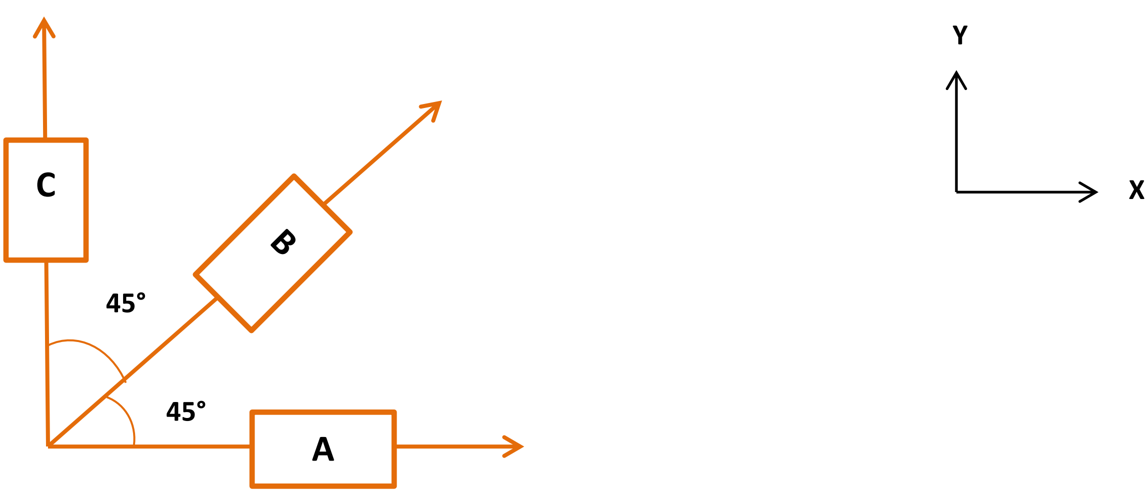
1. Rectangular strain gauge rosette

2. Delta strain gauge rosette

3. Star strain gauge rosette

Rectangular strain gauge rosette

A rectangular strain rosette consists of three strain gauges arranged as follows:-



We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

where ε_x = normal strain at a point in x-direction

ε_y = normal strain at a point in y- direction

γ_{xy} = shear strain at a point on x face in y direction

So, normal strain at $\theta = 0°$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0° + \frac{\gamma_{xy}}{2}\sin 0°$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x} \quad \text{\_\_\_\_\_\_\_\_\_\_} \quad \textbf{Eqn (1)}$$

Normal strain at $\theta = 45°$ (with respect to strain guage A, anticlockwise)

$$\varepsilon_B = (\varepsilon_n)_{\theta=45°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 90° + \frac{\gamma_{xy}}{2}\sin 90°$$

$$\varepsilon_B = (\varepsilon_n)_{\theta=45°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{\gamma_{xy}}{2}$$

$$\boldsymbol{\varepsilon_x + \varepsilon_y + \gamma_{xy} = 2\,\varepsilon_B} \quad \text{\_\_\_\_\_\_\_\_\_\_} \quad \textbf{Eqn (2)}$$

Similarly, normal strain at $\theta = 90°$ (with respect to strain guage A, anticlockwise)

$$\varepsilon_C = (\varepsilon_n)_{\theta=90°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 180° + \frac{\gamma_{xy}}{2}\sin 180°$$

$$\varepsilon_c = (\varepsilon_n)_{\theta=90^\circ} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) - \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_c = (\varepsilon_n)_{\theta=90^\circ} = \varepsilon_y} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad \textbf{Eqn (3)}$$

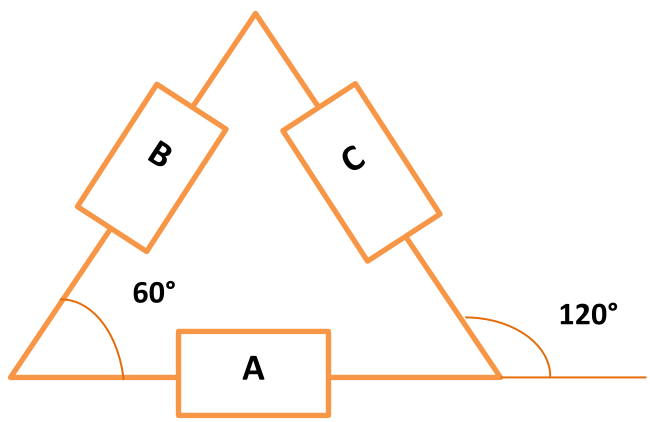
From Eqn 1 , 2 and 3

$$\boldsymbol{\gamma_{xy} = 2\,\varepsilon_B - \varepsilon_A - \varepsilon_c}$$

Note :- With the help of rectangular strain rosette we get strains in three arbitrary directions which in turn give us ε_x, ε_y **and** γ_{xy} and hence principal strains at a point on the surface of the object can be determined.

Delta strain gauge rosette

A delta strain gauge also consist of three strain gauges arranged as shown below



Delta strain gauge rosette

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0^\circ}}$$

$$\boldsymbol{\varepsilon_B = (\varepsilon_n)_{\theta=60^\circ}}$$

$$\boldsymbol{\varepsilon_c = (\varepsilon_n)_{\theta=120^\circ}}$$

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\text{Sin } 2\theta$$

So, normal strain at $\theta = 0°$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0° + \frac{\gamma_{xy}}{2}\text{Sin } 0°$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x} \quad \text{———————} \quad \textbf{Eqn (4)}$$

Normal strain at $\theta = 60°$ (with respect to strain gauge A, anticlockwise)

$$\boldsymbol{\varepsilon_B = (\varepsilon_n)_{\theta=60°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 120° + \frac{\gamma_{xy}}{2}\textbf{Sin } 120°} \quad \text{———————}$$

Eqn (5)

Similarly, normal strain at θ= 120° (with respect to strain gauge A , anticlockwise)

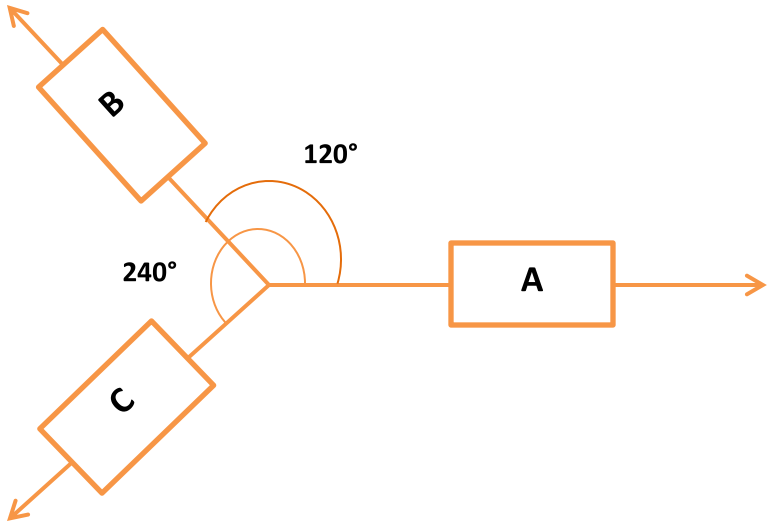
$$\boldsymbol{\varepsilon_C = (\varepsilon_n)_{\theta=120°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 240° + \frac{\gamma_{xy}}{2}\textbf{Sin } 240°} \quad \text{———————}$$

Eqn (6)

Similarly, $\boldsymbol{\varepsilon_x}$ **,** $\boldsymbol{\varepsilon_y}$ **and** $\boldsymbol{\gamma_{xy}}$ can be determined from equations 4, 5 and 6 and hence principal strains at a point on the surface of the object.

Star strain gauge rosette

This rosette consist of three strain gauges in three directions as shown below



Star strain gauge rosette

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°}$$

$$\varepsilon_B = (\varepsilon_n)_{\theta=120°}$$

$$\varepsilon_c = (\varepsilon_n)_{\theta=240°}$$

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\,\text{Sin}\ 2\theta$$

So, normal strain at $\theta = 0°$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0° + \frac{\gamma_{xy}}{2}\,\text{Sin}\ 0°$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x \qquad \text{———————} \qquad \textbf{Eqn (7)}$$

Normal strain at $\theta = 120°$ (with respect to strain gauge A, anticlockwise)

$$\varepsilon_B = (\varepsilon_n)_{\theta=120^\circ} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 240^\circ + \frac{\gamma_{xy}}{2}\sin 240^\circ$$ ———— **Eqn (8)**

Similarly, normal strain at θ= 240° (with respect to strain gauge A, anticlockwise)

$$\varepsilon_c = (\varepsilon_n)_{\theta=240^\circ} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 480^\circ + \frac{\gamma_{xy}}{2}\sin 480^\circ$$ ———— **Eqn (9)**

Similarly, by solving equations 7, 8 and 9 we can determine $\varepsilon_x, \varepsilon_y$ **and** γ_{xy} and hence principal strains at a point on the surface of the object.

Sample Question for Concept brush-up

S-1. Determine maximum shear stress at a point by using the following strain gauge readings of rectangular strain rosette

$\varepsilon_{0^\circ} = 400 * 10^{-6}$ **Young's Modulus = E= 200 GPa**

$\varepsilon_{45^\circ} = 375 * 10^{-6}$ **Poisson ratio = 0.25**

$\varepsilon_{90^\circ} = 200 * 10^{-6}$

Solution : We know from Eqn 1 ,2 and 3

$$\varepsilon_A = (\varepsilon_n)_{\theta=0^\circ} = \varepsilon_x = 400 * 10^{-6}$$

$$\varepsilon_c = (\varepsilon_n)_{\theta=90^\circ} = \varepsilon_y = 200 * 10^{-6}$$

$$\varepsilon_B = (\varepsilon_n)_{\theta=45^\circ} = 375 * 10^{-6}$$

and $\varepsilon_x + \varepsilon_y + \gamma_{xy} = 2\,\varepsilon_B$

$\gamma_{xy} = 150 * 10^{-6}$

Principal strain , $\varepsilon_{1,2} = \frac{1}{2}[(\varepsilon_x + \varepsilon_y) \pm \sqrt{(\varepsilon x - \varepsilon y)^2 + (\gamma xy)^\wedge 2}]$

$\varepsilon_1 = 425 * 10^{-6}$

$\varepsilon_2 = 175 * 10^{-6}$

Principal stress, $\sigma_1 = \frac{E}{1-\mu^2} (\varepsilon_1 + \mu \ \varepsilon_2)$

$\sigma_1 = 100$ Mpa

$\sigma_2 = \frac{E}{1-\mu^2} (\varepsilon_2 + \mu \ \varepsilon_1)$

$\sigma_2 = 60$ Mpa

Absolute $\tau_{max} = \frac{\sigma_1}{2}$ Or $\frac{\sigma_1 - \sigma_2}{2}$

As principal stresses like in nature therefore

Ans. **Absolute** $\boldsymbol{\tau_{max} = \frac{\sigma_1}{2} = 50}$ **Mpa**

References

1. Mechanics of Materials by B.C. Punmia

2. Strength of Materials by R.K. Bansal